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New Effect for Detecting Gravitational Waves by Amplification with Electromagnetic Radiation

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The perturbation of Dirac particles moving in a constant magnetic field is calculated for simultaneously incident parallel monochromatic circular polarized electromagnetic and gravitational waves. Resonances are found which depend on the initial energy of the charged particles, the magnetic field, and the frequencies of the incident waves. A suited choice of these parameters allows the selection of only one resonance that is proportional to the product of the squares of the amplitudes of both waves. This effect is valid for all bound systems of Dirac particles interacting simultaneously with electromagnetic and gravitational waves. At least in principle this resonance effect can be used to detect the gravitational waves in the lab. For regions of the universe with strong electromagnetic and gravitational waves and suited magnetic fields this effect may play another important part for the acceleration of charged particles.

1. INTRODUCTION

To search for gravitational waves in lab classical or quantum mechanical detectors can be used. Despite the experiments of Weber (1960, 1969) and many others (Abramovici *et al.*, 1992; Abramovici *et al.*, 1996; Braginskij *et al.*, 1972; Drever *et al.*, 1973; Levine and Garwin, 1973; Maischberger *et al.*, 1991; Tyson, 1973) and the theoretical calculations and estimations (Braginskij and Rudenko, 1970; Harry *et al.*, 1996; Schutz, 1997) gravitational waves have never been observed directly in lab. The reason for the difficulties of detecting gravitational waves is the small cross section of matter with gravitational waves which come from possible sources (Schutz, 1997) to us. The proposed or running experiments (Babusci *et al.*, 1997; Caldwell *et al.*, 1999; Schnier *et al.*, 1997; Shaddock *et al.*, 1998) try to avoid these difficulties by increasing the dimensions of the detectors. This way not only the effect of the gravitational waves is enlarged but expenses too.

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The motivation of this work was the search for a method to amplify the gravitational wave without amplifying competing phenomena. It seemed to be promising to look for a possibility to enlarge the effect of the gravitational waves on the detector at the "location" of the interaction of the gravitational radiation with the matter.

This paper is not intended to be an instruction for the construction of a gravitational wave detector. Rather it should demonstrate a "nihil obstat" to a method of gravitational wave detection that is different in principle.

2. GENERAL APPROACH

2.1. The Idea

We consider the well known bound system of a charged particle moving in a constant magnetic field. From Macedo and Nelson (1990) we know that perturbation of this system by gravitational waves causes resonance effects. On the other hand the structure of the general relativistic Dirac equation shows mixed terms of gravitational and electromagnetic fields. Therefore, the idea is to perturb a bound system by a gravitational *and* an electromagnetic wave at the same time with the ulterior motive to get a multiplicative selectable reaction of the bound system.

We exemplify first this idea in detail by considering an electron moving in a homogeneous constant magnetic field in x^1 direction which interacts with circular polarized monochromatic gravitational and electromagnetic waves propagating parallel to the magnetic field in x^1 direction. In the last sections we give a hint on the generalization to other bound systems.

2.2. Quantum Mechanical Formulation

In the linearized approximation to general relativity (Landau and Lifschitz, 1966; Misner *et al.*, 1973) for weak gravitational fields the metric tensor can be written

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon_{\mu\nu}, \qquad |\epsilon_{\mu\nu}| \ll 1 \tag{2.1}$$

with

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix} \tag{2.2}$$

(greek indices take the values 0, 1, 2, 3). By imposing the condition

$$\eta^{\alpha\beta}\left(\epsilon_{\alpha\nu} - \frac{1}{2}\eta_{\alpha\nu}\eta^{\kappa\lambda}\epsilon_{\kappa\lambda}\right)_{|\beta} = 0 \tag{2.3}$$

($A_{|\alpha}$ denotes the partial derivation of A with respect to α), the vacuum field equations are reduced to normal wave equations for the $\epsilon_{\mu\nu}$

$$\eta^{\alpha\beta}\epsilon_{\mu\nu|\alpha|\beta} = 0. \tag{2.4}$$

Solutions of (2.4) are, for example, monochromatic circular polarized waves (TT-gauge) in x^1 direction (according to the linearized approximation we neglect terms of ϵ^2 and higher in the following)

$$\begin{aligned}\epsilon_{22} &= \epsilon\cos[k(x^0 - x^1) + \alpha]\\ &= \frac{\epsilon}{2}\left[e^{ik(x^0-x^1)+i\alpha} + e^{-ik(x^0-x^1)-i\alpha}\right],\end{aligned} \tag{2.5a}$$

$$\begin{aligned}\epsilon_{23} &= \pm\epsilon\sin[k(x^0 - x^1) + \alpha]\\ &= \pm\frac{\epsilon}{2i}\left[e^{ik(x^0-x^1)+i\alpha} - e^{-ik(x^0-x^1)-i\alpha}\right],\end{aligned} \tag{2.5b}$$

($+/-$ left/right polarization, α arbitrary constant phase). With

$$k = \frac{\omega_G}{c} \tag{2.6}$$

(ω_G frequency of the wave).

The relativistic covariant Maxwell equations for the electromagnetic field $F^{\mu\nu}$ are (Adler *et al.*, 1965; Landau and Lifschitz, 1966)

$$F^{\mu\nu}{}_{\|\nu} = \frac{1}{\sqrt{-g}}\left(\sqrt{-g}F^{\mu\nu}\right)_{|\nu} = s^\mu, \tag{2.7a}$$

$$F_{[\mu\nu|\lambda]} \equiv F_{\mu\nu|\lambda} + F_{\lambda\mu|\nu} + F_{\nu\lambda|\mu} = 0 \tag{2.7b}$$

(s^μ four-current density, $g = \det\ g_{\mu\nu}$, $A_{\|\nu}$ denotes covariant derivation of A with respect to ν). Without charges and currents $s^\mu = 0$ and (2.7a) results to

$$\frac{1}{\sqrt{-g}}\left(\sqrt{-g}F^{\mu\nu}\right)_{|\nu} = 0 \tag{2.8}$$

while (2.7b) is unchanged.

The electromagnetic vector potential A^μ is correlated to the electromagnetic field tensor $F^{\mu\nu}$

$$F_{\mu\nu} = A_{\mu|\nu} - A_{\nu|\mu}. \tag{2.9}$$

For the following calculations we need the vector potential of a circular polarized monochromatic electromagnetic wave propagating in x^1 direction in linearized approximation. For the metric (2.5) the potential has the same form as in flat space and can be written

$$A_2 = A\ \cos[K(x^0 - x^1) + \beta]$$

$$= \frac{A}{2}\big[e^{iK(x^0-x^1)+i\beta} + e^{iK(x^0-x^1)-i\beta}\big], \tag{2.10a}$$

$$A_3 = \pm A\ \sin[K(x^0 - x^1) + \beta]$$

$$= \pm\frac{A}{2i}\big[e^{iK(x^0-x^1)+i\beta} - e^{-iK(x^0-x^1)-i\beta}\big], \tag{2.10b}$$

$$A_0 = A_1 = 0 \tag{2.10c}$$

and corresponding

$$A^2 = g^{2\mu}A_\mu = (-1 - \epsilon_{22})A_2 - \epsilon_{23}A_3, \tag{2.11a}$$

$$A^3 = g^{3\mu}A_\mu = -\epsilon_{23}A_2 - (1 - \epsilon_{22})A_3 \tag{2.11b}$$

($+/-$ left/right polarized, β arbitrary phase).

Additionally we need the vector potential A_μ for a constant homogeneous magnetic field in x^1 direction which also has the same form as in flat space for the metric (2.5)

$$A_\mu = \left(0, 0, \frac{H}{2}z, -\frac{H}{2}y\right). \tag{2.12}$$

This A_μ is Lorentz gauged.

The generalized relativistic Dirac equation (Brill and Wheeler, 1957) is

$$i\gamma^\mu\Psi_{\|\mu} - \frac{e}{\hbar c}\gamma^\mu A_\mu\Psi - \frac{mc}{\hbar}\Psi = 0, \tag{2.13}$$

where the generalized Dirac matrices γ^μ are calculated according to

$$\gamma^\mu = h^\mu_{(\alpha)}\gamma^{(\alpha)} \tag{2.14}$$

from the standard matrices $\gamma^{(\nu)}$ and the tetrads $h^\mu_{(\nu)}$ which are defined by the metric $g_{\mu\nu}$ of the gravitational field

$$h_{\mu(\alpha)}h_{\nu(\beta)}\eta^{(\alpha\beta)} = g_{\mu\nu}, \tag{2.15a}$$

$$h_{\mu(\alpha)}h^\mu_{(\beta)} = \eta_{(\alpha\beta)} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix} \tag{2.15b}$$

(indices in parenthesis are tetrad indices).

In order to solve Eq. (2.13) for the metric (2.5), the magnetic field (2.12) and the electromagnetic wave (2.10) we assume weak gravitational and electromagnetic waves which perturb the charged particle moving in the constant homogeneous magnetic field.

Therefore according to the perturbation theory (Sokolov and Ternov, 1968) we write (2.13) in the form

$$\left(i\hbar c\frac{\partial}{\partial x^0} - \mathfrak{H} - O\right)\Psi = 0 \tag{2.16}$$

where the unperturbed time independent Hamilton operator $\mathfrak{H}$ is defined with respect to (2.14) and (2.15) as

$$\mathfrak{H} = -i\hbar c\gamma^{(0)}\gamma^{(i)}\frac{\partial}{\partial x^i} + m_e c^2\gamma^{(0)} + e\gamma^{(0)}\gamma^{(2)}\frac{H}{2}z - e\gamma^{(0)}\gamma^{(3)}\frac{H}{2}y \tag{2.17}$$

and the perturbation operator O is written as

$$\begin{aligned} O = -\frac{1}{2}\hbar c\gamma^{(0)}\Bigg\{ & i\epsilon_{22}\gamma^{(2)}\frac{\partial}{\partial x^2} + i\epsilon_{23}\gamma^{(3)}\frac{\partial}{\partial x^2} + i\epsilon_{23}\gamma^{(2)}\frac{\partial}{\partial x^3} - i\epsilon_{22}\gamma^{(3)}\frac{\partial}{\partial x^3} \\ & - \frac{eH}{2\hbar c}z\left(\epsilon_{22}\gamma^{(2)} + \epsilon_{23}\gamma^{(3)}\right) + \frac{eH}{2\hbar c}y\left(\epsilon_{23}\gamma^{(2)} - \epsilon_{22}\gamma^{(3)}\right) \\ & - \frac{e}{\hbar c}\big[(2+\epsilon_{22})\gamma^{(2)}A_2 + \epsilon_{23}\gamma^{(3)}A_2 + \epsilon_{23}\gamma^{(2)}A_3 \\ & + (2-\epsilon_{22})\gamma^{(3)}A_3\big]\Bigg\}. \end{aligned} \tag{2.18}$$

H includes the influence of the magnetic field to the Dirac particle and O represents the influence of the gravitational and electromagnetic wave fields.

First we have to look for the exact solutions φ of the unperturbed system

$$\left(i\hbar c\frac{\partial}{\partial x^0} - \mathfrak{H}\right)\varphi = 0 \tag{2.19}$$

with the eigenfunctions

$$\varphi = e^{\frac{E_n x^0}{\hbar c}}\varphi_n(\vec{r}). \tag{2.20}$$

The time independent φ_n (n is a place holder for all quantum numbers) satisfy the eigenvalue equation

$$(E_n - \mathfrak{H})\varphi_n = 0 \tag{2.21}$$

and are to satisfy the orthonormality conditions

$$\int \varphi^{\dagger}_{n'}\varphi_n\, d^3x = \delta_{n',n}. \tag{2.22}$$

The solution of the Ψ of the Dirac equation (2.16) can be developed from the complete function system φ according to

$$\Psi = \sum_n c_n(x^0)\, e^{-i\,\frac{E_n x^0}{\hbar c}}\, \varphi_n(\vec{r}) \tag{2.23}$$

where $|c_n(x^0)|^2$ is the probability to find the Dirac particle at time t in state n, if we assume the same normalization as in (2.22)

$$\int \Psi^\dagger \Psi\, d^3x = 1 \quad \Rightarrow \quad \sum_n |c_n|^2 = 1. \tag{2.24}$$

Inserting (2.23) in (2.16), multiplying with $e^{i\,\frac{E_{n'}x^0}{\hbar c}} \varphi^\dagger_{n'}(\vec{r})$ and integration over the whole space results with respect to (2.22) in

$$i\hbar c \frac{\partial}{\partial x^0} c_{n'}(x^0) = \int d^3x\ \varphi^\dagger_{n'}(\vec{r})\, e^{i\,\frac{E_{n'}x^0}{\hbar c}} \sum_n O c_n(x^0)\, e^{i\,\frac{E_n x^0}{\hbar c}}\, \varphi_n(\vec{r}). \tag{2.25}$$

To solve (2.25) we use the well known iterative solution procedure. For this purpose we prepare the system that at $t = 0$ the system is in state |i> and all other states |f> are not occupied:

$$c_{\rm i}(0) = 1, \qquad c_{\rm f}(0) = 0. \tag{2.26}$$

Assuming that the perturbation O is small we can set in the first iteration step on the right side of (2.25)

$$c_{\rm i}(t) = c_{\rm i}(0) = 1; \tag{2.27}$$

then we get in first approximation for the solution of (2.25)

$$c_{\rm f}(x^0) = -\frac{i}{\hbar c} \int_0^{x^0} \int d^3x\ dx^0\ e^{i\,\frac{E_{\rm f}x^0}{\hbar c}}\, \varphi^\dagger_{\rm f}(\vec{r}) O\ e^{i\,\frac{E_{\rm i}x^0}{\hbar c}}\, \varphi_{\rm i}(\vec{r}) \tag{2.28}$$

and the transition probability $w_{\rm fi}$ per time unit from state $|i>$ to state $|f>$ is

$$w_{\rm fi} = c\frac{1}{x^0}(c^*_{\rm f} c_{\rm f}). \tag{2.29}$$

This perturbation calculation of first order breaks down if the perturbation is too big or the time period t of the perturbation is too long for the assumption (2.27) to be valid.

3. CALCULATION OF THE EFFECT

The exact solution of the unperturbed Dirac equation (2.19) with $\mathfrak{H}$ of (2.17) is (for another choice of the standard Dirac matrices $\gamma^{(\mu)}$ see Sokolov and Ternov, 1968)

$$\varphi_{p_1,n,m,\zeta} = \mathcal{C}\, e^{-i\frac{p_0 x^0}{\hbar}}\, e^{-i\frac{p_1 x^1}{\hbar}} u^{\frac{1}{2}b} \mathcal{H}^{\frac{1}{2}}\, e^{-\frac{u}{2}} \times \left\{ i\hbar\mathcal{H}^{\frac{1}{2}} \left[aF + 2u\,\frac{dF}{du} \right] e^{-ic\chi} u^{-\frac{1}{2}} \begin{pmatrix} -A \\ A \\ -B \\ B \end{pmatrix} + F\, e^{-id\chi} \begin{pmatrix} 1 \\ 1 \\ \mathcal{K} \\ \mathcal{K} \end{pmatrix} \right\}, \tag{3.1a}$$

where we have introduced the following abbreviations (the coordinate system may be chosen so that $eH > 0$)

$$\mathcal{H} = \frac{eH}{2\hbar c}, \tag{3.2a}$$

$$u = \mathcal{H} r^2, \tag{3.2b}$$

$$r = \sqrt{y^2 + z^2}, \qquad \chi = \arcsin \frac{y}{\sqrt{y^2 + z^2}}, \tag{3.2c}$$

$$A = \frac{p_1 + K(p_0 + m_e c)}{p_0^2 - m_e^2 c^2 - p_1^2}, \tag{3.2d}$$

$$B = \frac{p_0 - m_e c + p_1 K}{p_0^2 - m_e^2 c^2 - p_1^2}, \tag{3.2e}$$

$$m = \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots, \tag{3.2f}$$

$$a = \frac{1}{2} + |m| \mp \frac{1}{2} - m, \quad \text{upper/lower sign for } m \gtrless 0 \tag{3.2g}$$

$$b = |m| \mp \frac{1}{2}, \quad \text{upper/lower sign for } m \gtrless 0 \tag{3.2h}$$

$$c = m + \frac{1}{2}, \tag{3.2i}$$

$$d = m - \frac{1}{2}, \tag{3.2j}$$

$$F = {}_1F_1(-n, 1 + b, u), \quad \text{Laguerre polynomials (Abramowitz and Stegun, 1968)} \tag{3.2k}$$

$$\frac{E}{\hbar c} = \frac{p_0}{\hbar} = \pm\sqrt{4\mathcal{H}\left(n + \frac{1}{2}a\right) + \mu^2 + \frac{p_1^2}{\hbar^2}}, \quad \mu = \frac{m_e c}{\hbar}. \tag{3.2l}$$

$\mathcal{C}$ is a normalization constant which takes the value

$$\mathcal{C} = \left\{2\pi L \frac{\Gamma(1+b+n)}{n!}\left[(A^2+B^2)\left(p_0^2-\mu^2\hbar^2-p_1^2\right)+K^2+1\right]\right\}^{-\frac{1}{2}} \tag{3.3}$$

if we integrate over a volume with length L in x^1 direction. Taking into account that the wave functions φ are eigenfunctions of the following operators which commute with the Hamilton operator

(a) momentum operator in x^1 direction

$$-i\hbar\frac{\partial}{\partial x^1}\varphi_E = p_x\varphi_E, \tag{3.4}$$

(b) x^1 component of angular momentum

$$\begin{aligned}\mathcal{L}_x\varphi_E &\equiv \left[\frac{\hbar}{i}\left(y\frac{\partial}{\partial z}-z\frac{\partial}{\partial y}\right)+i\frac{\hbar}{2}\gamma^{(2)}\gamma^{(3)}\right]\varphi_E \\ &= \hbar m\varphi_E, \quad m = \pm\frac{1}{2}, \pm\frac{1}{2}, \ldots\end{aligned} \tag{3.5}$$

(c) operator of spin projection in the direction of propagation

$$M\varphi_E \equiv \sigma^{(i)}P_{(i)}\varphi_E = \hbar K_0\zeta\varphi_E, \quad \zeta = \pm 1, \tag{3.6}$$

with the kinetic energy

$$\hbar c K_0 = \hbar c\sqrt{\frac{p_0^2}{\hbar^2}-\mu^2}, \tag{3.7}$$

and

$$\sigma^{(i)} = \begin{pmatrix} -\gamma^{(2)}\gamma^{(3)} \\ \gamma^{(1)}\gamma^{(3)} \\ -\gamma^{(1)}\gamma^{(2)} \end{pmatrix}, \tag{3.8}$$

and

$$P_{(i)} = \begin{pmatrix} -i\hbar\frac{\partial}{\partial x^1} \\ -i\hbar\frac{\partial}{\partial x^2}+\frac{eH}{2c}z \\ -i\hbar\frac{\partial}{\partial x^3}-\frac{eH}{2c}y \end{pmatrix}, \tag{3.9}$$

we get for $\mathcal{K}$ the values

$$\mathcal{K} = \mp\frac{\sqrt{E^2/c^2-m^2c^2}}{E/c+mc} \quad \text{for } \zeta = \pm 1. \tag{3.10}$$

Additionally the following condition is satisfied

$$\int_V \varphi^\dagger_{p_1',n',m',\zeta'}\varphi_{p_1,n,m,\zeta}\, d^3x = \delta_{p_1',p_1}\delta_{n',n}\delta_{m',m}\delta_{\zeta',\zeta}. \tag{3.11}$$

In order to calculate the w_{fi} (2.29) with the perturbation operator (2.18) for the wave functions (3.1a) with the assumption (2.26) and the approximation (2.27) we use the cylindric coordinates (3.2b, 3.2c), some recursion and integral formulas for the Laguerre polynomials $L_b^n(u)$ (Abramowitz and Stegun, 1968) and assume that the wave functions φ are normalized in x^1 direction to the length L. Then we get after a lengthy but straightforward calculation

$$
\begin{aligned}
w_{\mathrm{fi}} = {} & \frac{2\pi\ e^2 H^2 \epsilon^2}{c} \mathcal{S}_{\mathrm{f}} \mathcal{S}_{\mathrm{i}} \Bigg\{ \mathcal{T}_{\mathrm{if}} N_{\mathrm{i}}(N_{\mathrm{i}}-1) \delta_{N_{\mathrm{f}},N_{\mathrm{i}}-2} \delta_{m_{\mathrm{f}},m_{\mathrm{i}}+2} \delta\left(\frac{E_{\mathrm{i}}-E_{\mathrm{f}}}{\hbar c} \pm k\right) \delta_{p_{1\mathrm{f}},p_{1\mathrm{i}} \pm \hbar k} \\
& + \mathcal{T}_{\mathrm{fi}}(N_{\mathrm{i}}+1)(N_{\mathrm{i}}+2) \delta_{N_{\mathrm{f}},N_{\mathrm{i}}+2} \delta_{m_{\mathrm{f}},m_{\mathrm{i}}-2} \delta\left(\frac{E_{\mathrm{i}}-E_{\mathrm{f}}}{\hbar c} \mp k\right) \delta_{p_{1\mathrm{f}},p_{1\mathrm{i}} \mp \hbar k} \Bigg\} \\
& + \frac{\pi\ e^3 H A^2}{\hbar c^2} \mathcal{S}_{\mathrm{f}} \mathcal{S}_{\mathrm{i}} \left\{ \mathcal{T}_{\mathrm{if}} N_{\mathrm{i}} \delta_{N_{\mathrm{f}},N_{\mathrm{i}}-1} \delta_{m_{\mathrm{f}},m_{\mathrm{i}}+1} \left[4\delta\left(\frac{E_{\mathrm{i}}-E_{\mathrm{f}}}{\hbar c} \begin{smallmatrix}+\\-\\-\\+\end{smallmatrix} K\right) \delta_{p_{1\mathrm{f}},p_{1\mathrm{i}} \begin{smallmatrix}+\\-\\-\\+\end{smallmatrix} \hbar K} \right. \right. \\
& + 4\epsilon\delta\left(\frac{E_{\mathrm{i}}-E_{\mathrm{f}}}{\hbar c} \begin{smallmatrix}+\\-\\-\\+\end{smallmatrix} K\right) \delta_{p_{1\mathrm{f}},p_{1\mathrm{i}} \begin{smallmatrix}+\\-\\-\\+\end{smallmatrix} \hbar K} \delta_{\begin{smallmatrix}k,+2K\\k,-2K\\k,+2K\\k,-2K\end{smallmatrix}} \cos\begin{pmatrix} +\alpha-2\beta \\ +\alpha+2\beta \\ -\alpha+2\beta \\ -\alpha-2\beta \end{pmatrix} \\
& \left. + \epsilon^2\delta\left(\frac{E_{\mathrm{i}}-E_{\mathrm{f}}}{\hbar c} \begin{matrix}+k-K\\+k+K\\-k+K\\-k-K\end{matrix}\right) \delta_{p_{1\mathrm{f}},p_{1\mathrm{i}} \begin{smallmatrix}\hbar(+k-K)\\\hbar(+k+K)\\\hbar(-k+K)\\\hbar(-k-K)\end{smallmatrix}} \right] \\
& + \mathcal{T}_{\mathrm{fi}}(N_{\mathrm{i}}+1) \delta_{N_{\mathrm{f}},N_{\mathrm{i}}+1} \delta_{m_{\mathrm{f}},m_{\mathrm{i}}-1} \left[4\delta\left(\frac{E_{\mathrm{i}}-E_{\mathrm{f}}}{\hbar c} \begin{smallmatrix}-\\+\\+\\-\end{smallmatrix} K\right) \delta_{p_{1\mathrm{f}},p_{1\mathrm{i}} \begin{smallmatrix}-\\+\\+\\-\end{smallmatrix} \hbar K} \right. \\
& + 4\epsilon\delta\left(\frac{E_{\mathrm{i}}-E_{\mathrm{f}}}{\hbar c} \begin{smallmatrix}-\\+\\+\\-\end{smallmatrix} K\right) \delta_{p_{1\mathrm{f}},p_{1\mathrm{i}} \begin{smallmatrix}-\\+\\+\\-\end{smallmatrix} \hbar K} \delta_{\begin{smallmatrix}k,+2K\\k,-2K\\k,+2K\\k,-2K\end{smallmatrix}} \cos\begin{pmatrix} -\alpha+2\beta \\ -\alpha-2\beta \\ +\alpha-2\beta \\ +\alpha+2\beta \end{pmatrix} \\
& \left. \left. + \epsilon^2\delta\left(\frac{E_{\mathrm{i}}-E_{\mathrm{f}}}{\hbar c} \begin{matrix}-k+K\\-k-K\\+k-K\\+k+K\end{matrix}\right) \delta_{p_{1\mathrm{f}},p_{1\mathrm{i}} \begin{smallmatrix}\hbar(-k+K)\\\hbar(-k-K)\\\hbar(+k-K)\\\hbar(+k+K)\end{smallmatrix}} \right] \right\},
\end{aligned}
\tag{3.12}
$$

Table I. Combinations of Polarization

	Sign ϵ_{23}	Sign A_3
1. Line	+	+
2. Line	+	−
3. Line	−	−
4. Line	−	+

with the following abbreviations

$$\mathcal{S}_k = \left[1 + \mathcal{K}_k^2 + \left(A_k^2 + B_k^2\right)\left(\frac{E_k^2}{c^2} - m_e^2 c^2 - p_{1_k}^2\right)\right]^{-1}, \tag{3.13a}$$

$$\mathcal{T}_{kr} = (B_k + \mathcal{K}_r A_k)^2. \tag{3.13b}$$

In the first brace of (3.12) the upper/lower sign denotes $+/-\epsilon_{23}$ (left/right circular polarization). In the second brace (3.12) the four lines denote the combinations of the polarization of the gravitational and electromagnetic waves (Table I). In (3.12) the abbreviations (3.2) are used and the definition

$$N_k \equiv n_k + \frac{1}{2}a_k \Rightarrow \frac{E_k}{\hbar c} = \pm\sqrt{4\mathcal{H}N_k + \mu^2 + \frac{p_{1_k}^2}{\hbar^2}}. \tag{3.14}$$

The indices k and r in the equations (3.13a), (3.13b), and (3.14) are place holders for the quantum numbers i before the beginning of the perturbation and f after the perturbation. It is clear from (3.12) that the transition probability consists of four terms: a pure gravitational term induced by the gravitational wave ($\sim\epsilon^2$), a pure electromagnetic term induced by the electromagnetic wave ($\sim A^2$), a first mixed term ($\sim\epsilon A^2$), and a second mixed term ($\sim\epsilon^2 A^2$). These latter two terms are the terms of interest since they represent the simultaneous interaction of *both*, gravitational and electromagnetic, waves with the bound system. The interaction of an electromagnetic wave alone with a charged particle in a constant magnetic field is well known (Redmond, 1965; Sokolov and Ternov, 1968). The interaction of a pure gravitational wave with a classical charged particle in a uniform magnetic field is calculated by Papadopoulos and Esposito (1981).

In reality neither the gravitational wave nor the electromagnetic wave have sharp frequencies $\omega_0 = ck_0$, but a frequency band width $\Delta\omega$ around ω_0, that is we have a wave packet with a Fourier spectrum. We assume that the intensities of the incident waves are constant within $\Delta\omega$ and the phases are incoherent. Then we can substitute

$$\epsilon^2\delta\left(\frac{E_\mathrm{i} - E_\mathrm{f}}{\hbar c} \pm k_0\right) \to u_{\mathrm{grav}}(k_0)\frac{16\pi G}{c^4} \times \int_{\Delta k} \frac{1}{k^2}\delta\left(\frac{E_\mathrm{i} - E_\mathrm{f}}{\hbar c} \pm k_0\right) dk, \tag{3.15}$$

where ϵ is the amplitude of the gravitational wave and $u(k_0)$ is the mean spectral energy density according to

$$u_{\text{grav}}\, dk = d\rho_{\text{grav}}.$$

Here ρ_{grav} is the mean energy density of the gravitational wave field. In the same manner we substitute

$$A^2\delta\left(\frac{E_{\rm i}-E_{\rm f}}{\hbar c} \pm K_0\right) \to u_{\text{em}}(K_0)\frac{4\pi}{c^2} \times \int_{\Delta K}\frac{1}{K^2}\delta\left(\frac{E_{\rm i}-E_{\rm f}}{\hbar c} \pm K_0\right) dK, \tag{3.16}$$

with

$$u_{\text{em}}\, dK = d\rho_{\text{em}},$$

where ρ_{em} is the mean energy density of the electromagnetic wave field.

For the mixed terms in (3.12) of gravitational and electromagnetic waves we substitute in an analogous manner

$$\epsilon A^2\delta\left(\frac{E_{\rm i}-E_{\rm f}}{\hbar c} \begin{smallmatrix}+\\-\\-\\+\end{smallmatrix} K\right)\delta_{\substack{k,+2K\\k,-2K\\k,+2K\\k,-2K}} \to \frac{4\pi}{c^2}\sqrt{\frac{16\pi G}{c^4}} \times \int_{\Delta k}\int_{\Delta K}\frac{1}{k}\frac{1}{K^2}\delta\left(\frac{E_{\rm i}-E_{\rm f}}{\hbar c} \begin{smallmatrix}+\\-\\-\\+\end{smallmatrix} K\right)\delta_{\substack{k,+2K\\k,-2K\\k,+2K\\k,-2K}} \times \sqrt{u_{\text{grav}}(k)}u_{\text{em}}(K)\, dk\, dK \tag{3.17}$$

and

$$\epsilon^2A^2\delta\left(\frac{E_{\rm i}-E_{\rm f}}{\hbar c} \begin{matrix}+k-K\\+k+K\\-k+K\\-k-K\end{matrix}\right) \to \frac{16\pi G}{c^4}\frac{4\pi}{c^2} \times \int_{\Delta k}\int_{\Delta K}\frac{1}{k^2}\frac{1}{K^2}\delta\left(\frac{E_{\rm i}-E_{\rm f}}{\hbar c} \begin{matrix}+k-K\\+k+K\\-k+K\\-k-K\end{matrix}\right) \times u_{\text{grav}}(k)u_{\text{em}}(K)\, dk\, dK. \tag{3.18}$$

The upper/lower sign in (3.15) and (3.16) denotes again left/right polarization and the four lines in (3.17) and (3.18) are explained in Table I.

4. DISCUSSION

4.1. The Various Terms of the Transition Probability

4.1.1. Pure Gravitational Transitions

The first term in (3.12) describes the transitions induced by the gravitational waves alone. Taking into account the abbreviations (3.14) and (3.2a) we see from (3.12) that we get transitions in this case only if

$$\delta_{N_{\rm f},N_{\rm i}-2}\delta\left(\frac{E_{\rm i}-E_{\rm f}}{\hbar c}\pm k\right)\delta_{p_{1{\rm f}},p_{1{\rm i}}\pm\hbar k}\neq 0. \tag{4.1}$$

This is true if

$$k=\frac{\omega}{c}=\mp\frac{2\,eH}{E_{\rm i}-cp_{1{\rm i}}}. \tag{4.2}$$

In this case we have also

$$\delta_{N_{\rm f},N_{\rm i}+2}\delta\left(\frac{E_{\rm i}-E_{\rm f}}{\hbar c}\mp k\right)\delta_{p_{1{\rm f}},p_{1{\rm i}}\mp\hbar k}\neq 0. \tag{4.3}$$

(upper/lower sign for left/right circular polarized gravitational waves). If the initial energy $E_{\rm i}$ and the initial momentum $p_{\rm i}$ of the charged particle in the direction of propagation of the gravitational wave are set, then the determination of the magnetic field filters the frequency (4.2) of the gravitational radiation. Only waves with this frequency cause transitions of the charged particle. In this case the transition probability is according to (3.12) and (3.15) with the abbreviations (3.13)

$$\begin{aligned} w_{\rm fi}=u_{\rm grav}(k_0)\frac{32\pi^2G\,e^2H^2}{c^5k_0^2}\mathcal{S}_{\rm f}\mathcal{S}_{\rm i}\big\{&\mathcal{T}_{\rm if}N_{\rm i}(N_{\rm i}-1)\delta_{N_{\rm f},N_{\rm i}-2}\delta_{m_{\rm f},m_{\rm i}+2}\delta_{p_{1{\rm f}},p_{1{\rm i}}\pm\hbar k_0}\\ &+\mathcal{T}_{\rm fi}(N_{\rm i}+1)(N_{\rm i}+2)\delta_{N_{\rm f},N_{\rm i}+2}\delta_{m_{\rm f},m_{\rm i}-2}\delta_{p_{1{\rm f}},p_{1{\rm i}}\mp\hbar k_0}\big\}, \end{aligned} \tag{4.4}$$

$$k_0=\mp\frac{2\,eH}{E_{\rm i}-cp_{1{\rm i}}}. \tag{4.5}$$

If the resonance condition (4.2) is fulfilled we have, from (4.1) and (4.3), energy changes of $\pm\hbar ck$, changes of momentum p_1 of $\pm\hbar k$ and, from (4.4), changes of the x^1 component of the angular momentum of $\pm 2\hbar$.

4.1.2. Pure Electromagnetic Transitions

Transitions caused by the electromagnetic waves alone are described by those terms in (3.12) which are proportional to A^2. In analogy to (4.1) we have the condition for these transitions

$$\delta_{N_{\rm f},N_{\rm i}-1}\delta\left(\frac{E_{\rm i}-E_{\rm f}}{\hbar c}\pm K\right)\delta_{p_{1{\rm f}},p_{1{\rm i}}\pm\hbar K}\neq 0. \tag{4.6}$$

This is true only if

$$K = \frac{\Omega}{c} = \mp \frac{eH}{E_{\mathrm{i}} - cp_{1\mathrm{i}}}. \tag{4.7}$$

In this case we have also

$$\delta_{N_{\mathrm{f}},N_{\mathrm{i}}+1}\delta\left(\frac{E_{\mathrm{i}} - E_{\mathrm{f}}}{\hbar c} \mp K\right)\delta_{p_{1\mathrm{f}},p_{1\mathrm{i}}\mp\hbar K} \neq 0, \tag{4.8}$$

(upper/lower sign corresponds to left/right circular polarized waves). If the magnetic field H, the initial energy E_{i}, and the initial momentum p_{i} are set, the filtered electromagnetic frequency (4.7) is one-half of the filtered gravitational frequency (4.2) (difference between dipol and quadrupol transitions). The transition probability according to (3.12) and (3.16) with the abbreviations (3.13) is

$$w_{\mathrm{fi}} = u_{\mathrm{em}}(K_0)\frac{16\pi^2 e^3 H}{\hbar c^4 K_0^2}\mathcal{S}_{\mathrm{f}}\mathcal{S}_{\mathrm{i}}\big\{\mathcal{T}_{\mathrm{if}}N_{\mathrm{i}}\delta_{N_{\mathrm{f}},N_{\mathrm{i}}-1}\delta_{m_{\mathrm{f}},m_{\mathrm{i}}+1}\delta_{p_{1\mathrm{f}},p_{1\mathrm{i}}\pm\hbar K_0}$$
$$+ \mathcal{T}_{\mathrm{fi}}(N_{\mathrm{i}}+1)\delta_{N_{\mathrm{f}},N_{\mathrm{i}}+1}\delta_{m_{\mathrm{f}},m_{\mathrm{i}}-1}\delta_{p_{1\mathrm{f}},p_{1\mathrm{i}}\mp\hbar K_0}\big\}, \tag{4.9}$$

$$K_0 = \mp\frac{eH}{E_{\mathrm{i}} - cp_{1\mathrm{i}}}. \tag{4.10}$$

The change of the x^1 component of the angular momentum is $\pm\hbar$ in contrast to the gravitationally induced transitions. For the changes of energy, momentum, and spin flip we have results analogous to the gravitational case.

4.1.3. The First Mixed Term of Gravitational and Electromagnetic Transitions

The first mixed term in (3.12) is proportional to ϵA^2. It is obvious that this term does not vanish in case of $k = \pm 2K$ only. The resonance condition is the same as in the previous case of pure electromagnetic waves, defined by (4.7), that is, the electromagnetic resonance condition (4.7) *and* the gravitational resonance condition (4.2) have to be fulfilled at the same time. In this case the transition probability according to (3.12) and (3.17) with the abbreviations (3.13) is

$$w_{\mathrm{fi}} = \frac{16\pi^2 e^3 H}{\hbar c^4}\sqrt{\frac{16\pi G}{c^4}}\mathcal{S}_{\mathrm{f}}\mathcal{S}_{\mathrm{i}}\left\{\mathcal{T}_{\mathrm{if}}N_{\mathrm{i}}\delta_{N_{\mathrm{f}},N_{\mathrm{i}}-1}\delta_{m_{\mathrm{f}},m_{\mathrm{i}}+1}\delta_{p_{1\mathrm{f}},p_{1\mathrm{i}}\begin{smallmatrix}+\\-\\-\\+\end{smallmatrix}\hbar K}\right.$$
$$\times\,\delta_{\begin{smallmatrix}k,+2K\\k,-2K\\k,+2K\\k,-2K\end{smallmatrix}}\cos\begin{pmatrix}+\alpha-2\beta\\+\alpha+2\beta\\-\alpha+2\beta\\-\alpha-2\beta\end{pmatrix}\int_{\Delta K}\int_{\Delta k}\frac{1}{k}\frac{1}{K^2}\sqrt{u_{\mathrm{grav}}(k)}u_{\mathrm{em}}(K)$$

$$\times\,\delta\left(\frac{E_{\mathrm{i}}-E_{\mathrm{f}}}{\hbar c}\begin{smallmatrix}+\\-\\-\\+\end{smallmatrix}K\right)\,dk\,dK+\mathcal{T}_{\mathrm{fi}}(N_{\mathrm{i}}+1)\delta_{N_{\mathrm{f}},N_{\mathrm{i}}+1}\delta_{m_{\mathrm{f}},m_{\mathrm{i}}-1}\delta_{p_{1\mathrm{f}},\,p_{1\mathrm{i}}\begin{smallmatrix}-\\+\\+\\-\end{smallmatrix}\hbar K}$$

$$\times\,\delta_{\begin{smallmatrix}k,+2K\\k,-2K\\k,+2K\\k,-2K\end{smallmatrix}}\cos\begin{pmatrix}-\alpha+2\beta\\-\alpha-2\beta\\+\alpha-2\beta\\+\alpha+2\beta\end{pmatrix}\int_{\Delta K}\int_{\Delta k}\frac{1}{k}\frac{1}{K^{2}}\sqrt{u_{\mathrm{grav}}(k)}\,u_{\mathrm{em}}(K)$$

$$\left.\times\,\delta\left(\frac{E_{\mathrm{i}}-E_{\mathrm{f}}}{\hbar c}\begin{smallmatrix}-\\+\\+\\-\end{smallmatrix}K\right)\,dk\,dK\right\}+w_{\mathrm{fi}}\text{ according to (4.9)}$$

$$+\,w_{\mathrm{fi}}\text{ according to (4.4).}\qquad(4.11)$$

4.1.4. The Second Mixed Term of Gravitational and Electromagnetic Transitions

The second mixed term in (3.12) is proportional to $\epsilon^2 A^2$. The resonance condition is

$$\delta_{N_{\mathrm{f}},N_{\mathrm{i}}-1}\delta\left(\frac{E_{\mathrm{i}}-E_{\mathrm{f}}}{\hbar c}\begin{smallmatrix}+k-K\\+k+K\\-k+K\\-k-K\end{smallmatrix}\right)\delta_{p_{1\mathrm{f}},\,p_{1\mathrm{i}}\begin{smallmatrix}\hbar(+k-K)\\\hbar(+k+K)\\\hbar(-k+K)\\\hbar(-k-K)\end{smallmatrix}}\neq 0.\qquad(4.12)$$

This is true only if

$$\begin{matrix}+k-K\\+k+K\\-k+K\\-k-K\end{matrix}=-\frac{eH}{E_{\mathrm{i}}-cp_{1\mathrm{i}}}.\qquad(4.13)$$

In this case we have also

$$\delta_{N_{\mathrm{f}},N_{\mathrm{i}}+1}\delta\left(\frac{E_{\mathrm{i}}-E_{\mathrm{f}}}{\hbar c}\begin{smallmatrix}-k+K\\-k-K\\+k-K\\+k+K\end{smallmatrix}\right)\delta_{p_{1\mathrm{f}},\,p_{1\mathrm{i}}\begin{smallmatrix}\hbar(-k+K)\\\hbar(-k-K)\\\hbar(+k-K)\\\hbar(+k+K)\end{smallmatrix}}\neq 0.\qquad(4.14)$$

The four lines correspond to the various polarizations of the circular electromagnetic and gravitational waves and are explained in Table I. The transition probability according to (3.12) and (3.18) with the abbreviations (3.13) is

$$w_{\mathrm{fi}}=\frac{64\pi^{3}G\,e^{3}H}{\hbar c^{8}}\mathcal{S}_{\mathrm{f}}\mathcal{S}_{\mathrm{i}}\left\{\mathcal{T}_{\mathrm{if}}N_{\mathrm{i}}\delta_{N_{\mathrm{f}},N_{\mathrm{i}}-1}\delta_{m_{\mathrm{f}},m_{\mathrm{i}}+1}\delta_{p_{1\mathrm{f}},\,p_{1\mathrm{i}}\begin{smallmatrix}\hbar(+k-K)\\\hbar(+k+K)\\\hbar(-k+K)\\\hbar(-k-K)\end{smallmatrix}}\right.$$

$$\times \int_{\Delta K}\int_{\Delta k} \frac{1}{k^2}\frac{1}{K^2} u_{\text{grav}}(k) u_{\text{em}}(K)\, \delta\left(\frac{E_{\text{i}} - E_{\text{f}}}{\hbar c} \begin{matrix} +k - K \\ +k + K \\ -k + K \\ -k - K \end{matrix}\right) dk\, dK$$

$$+ \mathcal{T}_{\text{fi}}(N_{\text{i}} + 1)\delta_{N_{\text{f}}, N_{\text{i}}+1}\delta_{m_{\text{f}}, m_{\text{i}}-1}\delta_{p_{1\text{f}}, p_{1\text{i}} \begin{matrix} \hbar(-k+K) \\ \hbar(-k-K) \\ \hbar(+k-K) \\ \hbar(+k+K) \end{matrix}}$$

$$\left.\times \int_{\Delta K}\int_{\Delta k} \frac{1}{k^2}\frac{1}{K^2} u_{\text{grav}}(k) u_{\text{em}}(K)\, \delta\left(\frac{E_{\text{i}} - E_{\text{f}}}{\hbar c} \begin{matrix} -k + K \\ -k - K \\ +k - K \\ +k + K \end{matrix}\right) dk\, dK \right\}. \quad (4.15)$$

The changes in energy, momentum, angular momentum, and spin orientation correspond to the changes of the pure electromagnetic case; this mixed term represents a gravitationally induced electromagnetic dipol transition. This term can be used to amplify electromagnetically the action of the gravitational wave. It may be mentioned here that the change of momentum p_{i} of, for example, $\hbar(k + K)$ *accelerates* the particle in x^1 direction.

4.2. The Effect Itself

If we want to detect a gravitational wave with a certain frequency ω it is obvious from the last section that we can choose a combination of E_{i}, p_{i}, H, and frequency Ω of the electromagnetic wave so that only (4.13) is satisfied but not (4.7). This way we can amplify the gravitational wave without having side effects of transitions induced by the electromagnetic wave without gravitational wave. In this case we measure transitions only if gravitational waves are present. In order to estimate the order of magnitude of this effect we introduce the following definitions

$$E_{\text{i}} = (1 + x)mc^2, \quad (4.16\text{a})$$

$$\hbar c\Delta = ymc^2, \quad (4.16\text{b})$$

where Δ is a combination of $\pm k \pm K$. For the sake of simplicity we assume

$$p_{1\text{i}} = 0 \quad (4.17)$$

and we set

$$\begin{aligned} S &:= \mathcal{T}_{\text{if}}\mathcal{S}_{\text{i}}\mathcal{S}_{\text{f}} \\ &\approx \mathcal{T}_{\text{fi}}\mathcal{S}_{\text{i}}\mathcal{S}_{\text{f}}, \end{aligned} \quad (4.18)$$

a quantity occurring in each term of (3.12). In the following estimations we assume

$$y \ll 1, \quad (4.19\text{a})$$

$$y \ll x, \quad (4.19\text{b})$$

which is justified for the examples as we will show later. In the *nonrelativistic* case, $x \ll 1$, we get for the quantity S in the case of no spin flips

$$S \sim \left(\frac{c}{mc^2}\right)^2. \tag{4.20}$$

With spin flip we have

$$S \sim \left(\frac{c}{mc^2}\right)^2 \frac{y^2}{x^2}. \tag{4.21}$$

In the *relativistic* case, $x \gg 1$, we have for the quantity S in the case of no spin flips

$$S \sim \left(\frac{c}{mc^2}\right)^2 \frac{1}{x^2}, \tag{4.22}$$

and with spin flip

$$S \sim \left(\frac{c}{mc^2}\right)^2 \frac{y^2}{x^4}. \tag{4.23}$$

It is annotated for exact calculations that S depends in all cases on the spin orientation. With (3.14) for $N_{\rm i}$, (3.18) for the elimination of the Delta-function, the resonance conditions (4.12), (4.13), (4.14), and the estimations (4.20)–(4.23) the terms of $w_{\rm fi}$ can be calculated from (3.12) or (4.15), respectively. So we get in the *nonrelativistic* case

$$w_{\rm fi}^{\uparrow\uparrow} \sim \frac{16\pi^2\, e^2\epsilon^2 S_{\rm em} x}{\epsilon_0 \hbar^2 c^2 k K^2}, \tag{4.24a}$$

$$w_{\rm fi}^{\uparrow\downarrow} \sim \frac{16\pi^2\, e^2\epsilon^2 S_{\rm em} y^2}{\epsilon_0 \hbar^2 c^2 k K^2 x} \tag{4.24b}$$

and in the *relativistic* case

$$w_{\rm fi}^{\uparrow\uparrow} \sim \frac{16\pi^2\, e^2\epsilon^2 S_{\rm em}}{\epsilon_0 \hbar^2 c^2 k K^2}, \tag{4.25a}$$

$$w_{\rm fi}^{\uparrow\downarrow} \sim \frac{16\pi^2\, e^2\epsilon^2 S_{\rm em} y^2}{\epsilon_0 \hbar^2 c^2 k K^2 x^2}, \tag{4.25b}$$

where $S_{\rm em}$ is the energy flux density of the electromagnetic wave and ϵ_0 is the dielectric constant.

4.3. Noise

There are some competing effects which could cover the gravitationally induced transitions. In the following sections we will consider the most important of them. Instrumental details of the detector are not taken into account.

4.3.1. Thermal Collisions

We assume a thermal Maxwell distribution of the velocities of the electrons for the temperature T. Then we have to estimate two probabilities.

First we have to calculate the probability that the energy increase or decrease of the electrons due to the thermal distribution is greater than the energy, for example, increase which we expect from the gravitationally induced transitions. This probability must be compared with the probability for these gravitationally induced transitions (and of course should be smaller). The energy difference of the gravitationally induced transitions according to (4.14) is approximately $\hbar\Omega$, where Ω is the frequency of the electromagnetic wave and the assumption was made that the frequency of the electromagnetic wave is much greater than the frequency of the gravitational wave. The corresponding transition probability w_{fi} is (4.15). The probability for electrons with energies greater than $\hbar\Omega$ is according to the well-known Maxwell distribution

$$w_1 = \frac{1}{\pi^{\frac{1}{2}}}\left(1 + 2\sqrt{\frac{\hbar\Omega}{kT}}\right) e^{-\frac{\hbar\Omega}{kT}}. \tag{4.26}$$

Second we have to calculate the probability that the energy increase or decrease of the electrons due to the thermal distribution is great enough to satisfy the resonance condition (4.7) for pure electromagnetic waves in the case where we have tuned the system to fulfill the resonance condition (4.13) for the mixed term only. The product of this probability w_2 with the transition probability for the pure electromagnetic case (4.9) must be compared with the probability for the gravitationally induced transitions w_{fi} (4.15) (and of course should be smaller). We get w_2 with the Maxwell distribution

$$w_2 = \frac{1}{\pi^{\frac{1}{2}}}\left(1 + 2\sqrt{\frac{\Delta}{kT}}\right) e^{-\frac{\Delta}{kT}}, \tag{4.27}$$

with

$$\Delta = \frac{\omega E_{\mathrm{i}}}{\Omega}. \tag{4.28}$$

ω is the frequency of the gravitational wave and E_{i} the initial energy of the electron.

4.3.2. Synchrotron Radiation

In case of high initial energy of the fermions the effect of synchrotron radiation will dominate the effect induced by the gravitational wave. The formulae for transitions caused by synchrotron radiation can be found in Sokolov and Ternov (1968, Chapter II). Therefore, if we want to detect the gravitational waves by the effect presented in this article we have to use nonrelativistic Dirac particles. It is remarkable that for energies

$$E \ll E_{1/2}, \tag{4.29}$$

with

$$E_{1/2} = mc^2 \left(\frac{mcR}{\hbar} \right)^{1/2}, \quad R = \sqrt{\frac{N}{\mathcal{H}}}, \quad N = 0, 1, \ldots \tag{4.30}$$

according to Sokolov and Ternov (1968, Chapter II, Section 5, Formula 5.19, p. 90) only transitions without spin flip occur.

For the estimation of the signal-to-noise ratio the transition probabilities induced by synchrotron radiation for both particles with relativistic and nonrelativistic kinetic energies are calculated according to Sokolov and Ternov (1968).

4.3.3. *Precision of the Initial Values*

The initial values

$$Q = \frac{eH}{E_{\mathrm{i}} - cp_{1\mathrm{i}}}. \tag{4.31}$$

must be precisely fixed in order to satisfy the resonance condition (4.13) for the gravitational case, but *not* the resonance condition (4.7) for pure electromagnetic waves. This limits the band width ΔQ to

$$\Delta Q < \frac{\omega}{c} \tag{4.32}$$

or

$$\frac{\Delta Q}{Q} < \frac{\omega}{\Omega + \omega} \tag{4.33}$$

(ω, Ω are frequencies of the gravitational and electromagnetic wave).

4.3.4. *Homogeneity of the Magnetic Field*

Inhomogeneities of the magnetic field shift in first approximation the energy according to (3.14)

$$\frac{\Delta E}{\hbar c} = \pm \sqrt{2 \frac{e(H + \Delta H)}{\hbar c} N + \mu^2 + \frac{p_1^2}{\hbar^2}}. \tag{4.34}$$

Therefore, in addition to the condition (4.33) of the previous section which limits the inhomogneity of the magnetic field for other reasons, the following condition for ΔH must be valid in order to avoid energy shifts greater than the expected energy increase/decrease induced by the gravitational and electromagnetic waves of the order $\hbar\Omega$ (4.14) (where Ω is the frequency of the electromagnetic wave and the assumption holds $\Omega \gg \omega$)

$$\hbar c \sqrt{2\,\frac{e(H+\Delta H)}{\hbar c} N + \mu^2 + \frac{p_1^2}{\hbar^2}} < \hbar c \sqrt{2\,\frac{eH}{\hbar c} N + \mu^2 + \frac{p_1^2}{\hbar^2}} + \hbar\Omega. \tag{4.35}$$

This leads to the condition

$$\frac{\Delta H}{H} < \hbar\Omega \frac{2E_{\mathrm{i}} + \hbar\Omega}{E_{\mathrm{i}}^2 - m^2c^4}. \tag{4.36}$$

4.3.5. Strong Electromagnetic Waves

In order to amplify the gravitational waves by electromagnetic waves we need high energy densities of the electromagnetic field because of the extremely small amplitudes of the expected gravitational waves. Therefore, the terms proportional $A^2,\ \epsilon A^2,\ \epsilon^2 A^2$ in (3.12) are no more of the same order of magnitude and, therefore, the perturbation calculation is not consistent. For a consistent perturbation calculation we had to use the exact solution of the Dirac equation for a constant magnetic field *and* an electromagnetic wave (Redmond, 1965) to calculate the scattering of the gravitational wave. Because of the complexity of the exact solution an unproportionally high expenditure would be necessary for these calculations. For strong electromagnetic waves the assumption (2.27) of the perturbation calculation is not true. Therefore, the resonance conditions (4.6) are no more exactly valid. When the magnetic field and the initial energy of the particle are chosen in order to satisfy the resonance condition (4.12) for a certain frequency of a electromagnetic and gravitational wave and thus to generate transitions proportional $\epsilon^2 A^2$, this amplification effect could be overlapped by transitions proportional A^2 caused by the strong electromagnetic waves out of the resonance frequency (4.7). According the calculations of Redmond (1965, p. 1169, Formula 5.32) the amplification effect can be used for the detection of the gravitational waves better, more for the functions R, K, J (formulae 3.10, 3.12, 5.12 of Redmond, 1965) *out of the resonance* the following conditions are satisfied

$$|R| \ll 1, \tag{4.37a}$$

$$\frac{|K|}{\sqrt{\frac{\hbar c}{eH}}} \ll 1, \tag{4.37b}$$

$$|J| \ll 1. \tag{4.37c}$$

If we assume for the electromagnetic wave

$$A(\omega_1, \omega_2, t) \sim A \int_{\omega_1}^{\omega_2} a(\omega)\Theta\left(-\frac{T}{2}, +\frac{T}{2}\right)\, e^{i\omega t}\, d\omega, \tag{4.38}$$

these restrictions (4.37) lead to the conditions (the resonance frequency ω_0 is out of the interval (ω_1, ω_2))

$$1 \gg \left| \frac{eA}{mc^2} \bar{\omega} a(\bar{\omega}) \left(\int_0^{(\omega_2-\omega_0)\frac{T}{2}} \frac{\sin y}{y}\, dy - \int_0^{(\omega_1-\omega_0)\frac{T}{2}} \frac{\sin y}{y}\, dy \right) \right|, \quad \text{(4.39a)}$$

$$\sqrt{\frac{\hbar c}{eH}} \gg \left| \frac{eA}{mc} a(\bar{\omega}) \left(\int_0^{(\omega_2-\omega_0)\frac{T}{2}} \frac{\sin y}{y}\, dy - \int_0^{(\omega_1-\omega_0)\frac{T}{2}} \frac{\sin y}{y}\, dy \right) \right|, \quad \text{(4.39b)}$$

$$1 \gg \left| \frac{e^2 A^2 a^2(\bar{\omega})}{\hbar mc} \left(\frac{\omega_1+\omega_2}{c} \int_0^{(\omega_1+\omega_2)\frac{T}{2}} \frac{\sin y}{y}\, dy - \frac{\omega_1}{c} \int_0^{\omega_1 T} \frac{\sin y}{y}\, dy \right.\right.$$
$$\left. - \frac{\omega_2}{c} \int_0^{\omega_2 T} \frac{\sin y}{y}\, dy \right) + \frac{c}{\hbar m} \left(\frac{e^2 AHa(\bar{\omega})}{mc^2(\omega_0 - \bar{\omega})} \right)^2 \left(\frac{2\omega_0 - \omega_1 - \omega_2}{c} \right.$$
$$\times \int_0^{(2\omega_0-\omega_1-\omega_2)\frac{T}{2}} \frac{\sin y}{y} dy - \frac{\omega_0 - \omega_1}{c} \int_0^{(\omega_0-\omega_1)\frac{T}{2}} \frac{\sin y}{y}\, dy$$
$$\left.\left. - \frac{\omega_0 - \omega_2}{c} \int_0^{(\omega_0-\omega_2)\frac{T}{2}} \frac{\sin y}{y}\, dy, \right) \right| \quad \text{(4.39c)}$$

where ω_0 is the resonance frequency according to (4.13), (4.15) and $\bar{\omega} = \frac{\omega_1+\omega_2}{2}$. It was assumed that $a(\omega)$ is approximately constant in the interval (ω_1, ω_2). Therefore, the amplification effect is usable in principle if

$$\hat{\omega} T \gg 1, \quad \text{(4.40)}$$

($\hat{\omega} = \omega_1 - \omega_0, \omega_2 - \omega_0, \ldots$ according to the upper limit of the integrals in (4.39). This determines a lower limit for the interaction time of the electromagnetic wave with the Dirac particle and thus a lower limit for the duration to keep stable the system of fermions, magnetic field, and electromagnetic wave.

4.3.6. Gravitational Radiation of the Electromagnetic Field

Generally it is not necessary to take into consideration in the linear approximation of the gravitational field the gravitational radiation of the electromagnetic field if the energy densitiy of the electromagnetic field does not exceed the magnitude of the energy density of the gravitational wave itself (in this approximation the gravitational action of the gravitational field is also neglected). But even if the energy density of the electromagnetic wave is higher than the energy density of the gravitational wave (with frequency ω) it can be determined with the help of the resonance conditions (4.1), (4.6), and (4.12) which kind of gravitational field caused the transitions if the gravitational field induced by the electromagnetic wave has a different frequency Ω (the gravitational field of an electromagnetic wave, Ketsaris, 1974).

4.3.7. Line Width and Travel Time

The finite life time τ of the energy levels of the electron causes, according to the uncertainty relation, a broadening ΔE of the energy levels

$$\tau \Delta E > \hbar. \tag{4.41}$$

This line width ΔE must of course be smaller than the energy difference of the two resonance conditions (4.7) and (4.13) since we want to exclude transitions induced by pure electromagnetic waves while including those of gravitational *and* electromagnetic waves. Therefore, we want

$$\Delta E < \hbar\omega, \tag{4.42}$$

with ω the frequency of the gravitational wave.

On the one hand the finite life time τ is correlated to the transition probability w

$$\tau = \frac{1}{w}. \tag{4.43}$$

As long as the transition probabilities of the competing effects of the previous sections are smaller than the probability of the gravitationally induced transitions (4.15) which is necessary for a detection of the effect, the relation (4.42) is valid.

On the other hand the finite life time τ is limited by the duration of the measurement or by the travel time T_{tr} of the electron. With (4.41) and (4.42) this leads to a first lower limit for T_{tr}

$$T_{\mathrm{tr}} > \omega^{-1}. \tag{4.44}$$

But the finite travel time causes violations of the resonance conditions (4.7) and (4.13) due to the fact that we have to replace the integration with respect to time from $-\infty$ to $+\infty$ by the interval, for example, $\frac{-T_{\mathrm{tr}}}{2}, \frac{+T_{\mathrm{tr}}}{2}$. Therefore, we have transitions out of the resonance since the Delta-functions of (3.12) are no more valid. We must estimate the probability $w_{\mathrm{em_{out}}}$ for transitions induced by pure electromagnetic waves while the initial values were chosen to satisfy (4.13). This out-of-resonance probability which violates the resonance condition (4.6) and (4.8) by the amount of $\hbar\omega$ is greater than zero since due to the finite integration time T_{tr} the conditions (4.6) and (4.8) are no more strictly valid. Finally $w_{\mathrm{em_{out}}}$ must be compared with the probability w_{fi} (4.15) for gravitationally induced transitions and of course must be smaller. This fixes a more serious second lower limit for the travel time T_{tr}.

Table II. Numerical Values for the Amplification of Gravitational Waves by Electromagnetic Radiation

x	Ω (Hz)	S_{em} (W/m^2)	$w^{\uparrow\uparrow}$ (1/s)	$w^{\uparrow\downarrow}$ (1/s)	SNR	H (Gauss)	R (m)	$\frac{\Delta\Omega}{\Omega}$	$\frac{\Delta Q}{Q}$	$\frac{\Delta H}{H}$	y
10^{-6}	10^{5}	10^{6}	10^{-6}	10^{-26}	$\ll 10^{0}$	10^{-2}	10^{-2}	10^{-3}	10^{-2}	10^{-10}	10^{-16}
10^{+3}	10^{5}	10^{6}	10^{0a}	10^{-38}	$\ll 10^{0}$	10^{0}	10^{+4}	10^{-3}	10^{-2}	10^{-19}	10^{-16}
10^{-3}	10^{10}	10^{6}	10^{-13}	10^{-29}	$\geq 10^{1}$	10^{+3}	10^{-4}	10^{-8}	10^{-7}	10^{-8}	10^{-11}
10^{-6}	10^{13}	10^{15b}	10^{-13}	10^{-17}	$\ll 10^{0}$	10^{+6}	10^{-10}	10^{-11}	10^{-10}	10^{-2}	10^{-8}
10^{-1}	10^{13}	10^{9c}	10^{-14}	10^{-28}	$\ll 10^{0}$	10^{+6}	10^{-5}	10^{-11}	10^{-10}	10^{-6}	10^{-8}
10^{+3}	10^{13}	10^{7d}	10^{-15}	10^{-37}	$\ll 10^{0}$	10^{+9}	10^{-4}	10^{-11}	10^{-10}	10^{-11}	10^{-8}

Note: The entries are explained in Section 4.4.
[a] This is only a theoretical value. In reality the interaction time t must be much smaller to be consistent with the perturbation theory.
[b] A laser with a power of 10^3 W is assumed. This power is focused on an area of $(10^4 \times R)^2$.
[c] A laser with a power of 10^3 W is assumed. This power is focused on an area of $(10^2 \times R)^2$.
[d] A laser with a power of 10^3 W is assumed. This power is focused on an area of $(10^1 \times R)^2$.

4.4. Quantification of the Effect

In Table II we give numerical values for some cases of amplification of gravitational waves by electromagnetic waves. We assume that the incident electromagnetic wave, the gravitational wave, and the magnetic field are parallel and the waves are approximately monochromatic and circular polarized. For the computation of the transition probabilities w_{fi} (4.15) of an electron we assume furthermore (4.17). For x and y the definitions (4.16) are used. For the gravitational radiation we assume a wave with frequency $\omega_{\mathrm{grav}} = 10^3$ Hz, an amplitude $\epsilon = 10^{-22}$, and a pulse time $t_{\mathrm{grav}} = 10^{-3}$ s. A possible source of this gravitational radiation could be the birth of a neutron star in the Virgo cluster (Schutz, 1997). R in Table II is

$$R = \sqrt{\frac{2N\hbar c}{eH}} \tag{4.45}$$

the mean radius of the electron in the magnetic field H. $w^{\uparrow\uparrow}$ or $w^{\uparrow\downarrow}$ respectively are the transition probabilities without or with spin flip (these values are proportional to ϵ^2). These quantities are in per second and have to be multiplied by the interaction time t_{grav} to give the transitions. Ω is the mean frequency of the electromagnetic wave, $\Delta\Omega$ is the bandwidth of the electromagnetic wave, and S_{em} its energy flux density per time and surface unit. With

$$Q = \frac{eH}{E_{\mathrm{i}} - cp_{1\mathrm{i}}} \tag{4.46}$$

$\frac{\Delta Q}{Q}$ indicates the necessary accuracy of keeping Q constant in order to exclude transitions (4.9) with the resonance condition (4.7) caused by the pure electromagnetic wave with frequency Ω while those of the mixed terms (4.15) with resonance condition (4.13) and frequency $\Omega + \omega$ are active. $\frac{\Delta H}{H}$ indicates the necessary homogeneity of the magnetic field to detect the energy increase/decrease of approximately $\hbar\Omega$ of the electrons. The signal-to-noise ratio (SNR) is the ratio of the transition probability $w^{\uparrow\uparrow}$ induced by the interaction with the electromagnetic and gravitational wave to the transition probability caused by the competing effects (noise). Here we have assumed a temperature of 10^{-3} K and a travel time of the electrons of 10^6 s.

If we want to detect gravitational waves with frequency ω we can choose several combinations of initial values of the electron energy E_{i} and the frequency of the electromagnetic wave Ω where in *absence* of the gravitational wave neither the resonance condition (4.13) for mixed terms nor the resonance condition (4.7) for pure electromagnetic waves is satisfied but *with* incident gravitational waves the resonance condition (4.13) is fulfilled, but *not* the resonance condition (4.7). The latter case for the presence of the gravitational wave then determines the necessary magnetic field H. In case of transition the electron absorbes or emittes approximately the amount $\hbar\Omega$ of energy. The total absorbed energy ΔE is the difference of the product of the probability for absorbing this amount and the product of the probability for emitting this amount.

The rows in Table II show the data for some combinations of initial values for the energy of the electron (indicated by x according (4.16a)), the frequency Ω, and the energy flux density S_{em} of the electromagnetic wave (for lasers and its data cf. e.g., Brunner and Junge, 1989), where the quantities $w^{\uparrow\uparrow}$, $w^{\uparrow\downarrow}$, H, and R are calculated from the initial values, and SNR is calculated from the initial values with respect to the noise effects and the assumption that $\frac{\Delta\Omega}{\Omega}$, $\frac{\Delta Q}{Q}$, and $\frac{\Delta H}{H}$ are limited by the indicated values. Furthermore, we assume for the SNR that S_{em} does vary less than 1% (photon shot noise is far below this limit). y is shown in the table to demonstrate the validity of (4.19a) and (4.19b).

The acceleration of the electrons in x^1 direction is for all rows

$$\Delta p_{\mathrm{i}} = \frac{\Delta E}{c} \tag{4.47}$$

where ΔE is the energy absorbed by the electron from the waves.

Row 3 of Table II indicates the combination of electron initial energy, frequency and power of the amplifying electromagnetic wave, and signal-to-noise ratio where the detection of gravitational waves could be possible. The SNR in this case is slightly better than the SNR of LIGO II or LISA in the gravitational wave frequency range of 10^3 Hz. In all other cases the noise mainly by thermal collisions according to (4.26) and (4.27) predominates the effect.

Of course for the high energies of the electromagnetic field used in the examples of Table II we had to consider higher orders of the electromagnetic wave amplitude A in the perturbation calculation to be consistent with the order of magnitude of the gravitational wave (for the highest energies in Table II we had to go up to the 10th order of the electromagnetic terms), but for a suited choice of the frequencies Ω, ω, and the magnetic field H all these additional frequencies do not satisfy the resonance condition.

4.5. Generalization of the Effect

First we see from (4.24) and (4.25) that the effect is proportional to e^2. Therefore, instead of electrons heavy ions could be examined with respect to their suitability for the detection of gravitational waves.

Second when we have electromagnetic *and* gravitational waves the structure of the generalized Dirac equation (2.13) implies always mixed terms of combinations of the frequencies and amplitudes of both waves in the transition probability. Therefore, whenever we have a bound system with resonance effects when interacting with a electromagnetic wave, this system in principle can be used for the detection of gravitational waves because of the occurrence of *multiplicative* terms of the *amplitudes* of the electromagnetic and gravitational waves and the *different* resonance conditions for the pure electromagnetic wave and the mixture of electromagnetic and gravitational wave. In all these cases the parameters of the bound system can be chosen so that resonance will only occur in the presence of both a gravitational and the electromagnetic wave but not in the case of absence of the gravitational wave.

For example, the bound system of electrons in an atom could be taken into account for the detection of gravitational waves. Besides the well-known resonance frequencies for the excitation of the electrons by electromagnetic waves there are slightly different resonance frequencies in the presence of electromagnetic *and* gravitational waves and the transition probabilities for these resonance conditions are proportional to the product of the amplitudes of both waves. Of course the line broadening of the electron levels will complicate the experimental prove of the existence of gravitational waves with this effect, but in principle this or other bound systems can be checked for their suitability for a detection of gravitational waves on earth.

5. CONCLUSION

A bound system of Dirac particles interacting with a plane gravitational and electromagnetic wave has resonances corresponding to the frequency of the electromagnetic wave alone, to the frequency of the gravitational wave alone, and to a combination of the frequencies of both waves. The resonance effect for the latter case is proportional to the product of the strength of the two fields. A suited combination of the parameters of the bound system and the frequencies of the two waves allows to select only this resonance case.

We have exemplified this effect in the previous sections for the bound system of an electron moving in a constant magnetic field which interacts with monochromatic circular polarized plane electromagnetic and gravitational waves. The quantification of the effect for a gravitational wave generated, for example, by the birth of a neutron star in a neighbouring galaxy amplified by an electromagnetic wave allows for a suited choice of the parameters as shown in Table II in principle the detection of the gravitational wave. The SNR for a gravitational frequency range of 10^3 Hz is slightly better than that of LIGO II and LISA in this range. The relation of the transition probability $w_{\mathrm{fi(em+grav)}}$ in case of the presence of both waves to the transition probability $w_{\mathrm{fi(grav)}}$ caused by the gravitational wave alone is proportional to the quotient of the energy flux density S_{em} of the electromagnetic wave and the product of the magnetic field H and the square of the frequency Ω of the electromagnetic wave

$$\frac{w_{\mathrm{fi(em+grav)}}}{w_{\mathrm{fi(grav)}}} = \frac{S_{\mathrm{em}}}{H\Omega^2} \tag{5.1}$$

Of course it is mentioned that competing effects may impede an experimental result.

The calculations in this article rely on weak gravitational waves. It can be assumed that in regions of the universe with strong electromagnetic waves *and* strong gravitational waves and suited magnetic fields the occurrence of terms with the product of the amplitudes of both waves in the Dirac equation (2.16) plays an important part for the acceleration of charged particles. But this case which is beyond the perturbation calculation is for further research.

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